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Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, September 2014
(2008 Scheme)
(Special Supplementary)
08.303 : DISCRETE STRUCTURES (R F)

Time : 3 Hours Max. Marks : 100

PART – A

(Answer **all** questions)

1. Give the truth table for the formula $\neg(\neg P \wedge \neg Q)$

2. Show that $(\exists x) M(x)$ follows logically from the premises $(x)(H(x) \rightarrow M(x))$ and $(\exists x) H(x)$.

3. Explain free and bound variables with examples.

4. Define partition of a set. Give an example.

5. What do you mean by composition of binary relations ?

6. What is a bijective mapping ? Give an example.

7. Show that the set of integers, positive, negative and zero is denumerable.

8. Define a monoid with an example.

9. Explain path matrix of a graph with an example.

10. Define integral domain. Give an example. **(10×4=40 Marks)**

PART – B

Module – 1

11. a) Prove without using truth table

$P \rightarrow (Q \wedge R) \rightleftarrows (P \wedge \neg Q) \rightarrow R$. 10

b) Show that from

i) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$ and

ii) $(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows. 10

OR

12. a) Show that the following statements constitute a valid argument : "If A works hard, then either B or C will enjoy themselves. If B enjoy himself, then A will not work hard. If D enjoy himself, then C will not. Therefore, if A works hard, then D will not enjoy himself. 10

b) Give a direct and an indirect proof of the implication :

$p \rightarrow q, q \rightarrow r, \neg(p \wedge r), p \vee r \Rightarrow r$. 10

Module – 2

13. a) The harmonic numbers H_k, k = 1, 2, 3 ... are defined by

$$H_k = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{k}$$

Use mathematical induction to show that $H_2n \geq 1 + \frac{n}{2}$, whenever n is a non negative integer. 10

b) Let f, g, h : R → R be given by

f(x) = x + 2, g(x) = x – 2, h(x) = 3x

Find fog, foh, gof, goh, hof, hog and fohog. 10

OR

14. a) Show that a finite subset of a denumerable set is also denumerable. 10

b) Define a primitive recursive function. Show that the function f(x, y) = x + y is primitive recursive. 10

Module – 3

15. a) State and prove Lagrange's theorem. 10

b) Let * be the binary operation defined on the set of all positive rational numbers Q^+ by

$$a * b = \frac{ab}{2}$$

Show that Q^+ is an Abelian group under this operation. 10

OR

16. a) Show that in a lattice if $a \leq b \leq c$, then $a \oplus b = b * c$ and

$(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$. 10

b) Show that every finite integral domain is a field. 10
